Analyses ✓

Phil. 12B

Exercise on Derivations

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(For each of the following arguments derive the conclusion from the premises, if any. Use only arguments of the kind indicated.

(a) Umd

$(w)(x)(y)(z)((\exists y)Fxyz \vee (z)Gywz)$

$((x)(z)(\exists y)Fxyz \vee (w)(y)(z)Gywz)$

(b) Ued, Tb

$(x)Hax$

$(x)(Fx \vee (w)Gwx)$

$(x)(-Hax \vee -Hxb \vee -Fx)$

$(x)Hxb$

Gcd

(c) Ued

$(z)(y)(x)Fxyz$

$Fuxy$

(d) Ued

$(z)(y)(x)Fxyz$

$Fzzx$

(e) Ued

$(z)(y)(x)Fxyz$

$Fyzz$

(f) Ued, Tb

$(x)(y)(Gxy \rightarrow Hxy)$

$(x)(y)(Fxy \rightarrow Gxy)$

$(x)Fxa$

$(x)Hxa$

(g) Ued, Tb

$(x)(y)(z)((Lxy \,\&\, Lyz) \rightarrow Lxz)$

$(x)(y)(z)-Lxx$

$(x)(y)(Lxy \rightarrow -Lyx)$

(h) Ued, Tb

$(x)(y)Fxy$

$(x)(Gx \rightarrow Hxx)$

$(x)(y)-Hxy$

$(x)((y)Fxy \rightarrow Gx)$

$(x)Jx$

(i) Ued, Tb

$-(x)Fx \vee Fa$

(j) Ued, Tb

$(x)((y)Hxy \rightarrow Hxx)$

(k) Ued, Umd

$(w)(x)(y)(z)(Fwxy \vee Gxyz)$

$((w)Fwxy \vee (z)Gxyz)$

(l) Ued, Tb

$(y)(Fx \,\&\, Gy)$

$(Fx \,\&\, (y)Gy)$

(m) Umd, Tb

$(y)(Fx \,\&\, Gy)$

$(Fx \vee (y)Gy)$

(n) Umd, Tb

$(y)(Fx \rightarrow (Gx \rightarrow Hy))$

$(Fx \rightarrow (Gx \rightarrow (y)Hy))$

(o) Umd, Tb

Fx

$(y)Fx$

I. 1. (x)Fx v (∃x)-Fx valid by redux tree Faj=T or Faj=F
2. (∃x)Fx intuitively contingent
3. (x)(Fx v - Fx) valid
4. (x)Fx v (x)-Fx
5. -(x)Fx v -(x)-Fx
6. -(x)Fx → (x)Fx
7. (x)(Fx → -Fx)
8. (∃x)(Fx ↔ -Fx)

II. 9. -(Fa & -Fa)
10. (x)Fx → Fa 'If x is F then a is F'
11. Fa → (x)Fx
12. Fa & -(∃x)Fx contingent F: eat a: minn b: costello
13. Fa v -(∃x)Fx contingent

III. 14. (x)(Fx → Gx) contingent
15. (∃x)(-Fx & -Gz) contingent
16. (x)(Fx v Gx) & (x)(-Fx v -Gx) contingent
17. (x)((-Fx v -Gx) v (Fx & Gx)) contingent
18. (∃x)(Fx & Gx) & -(∃x)Gx
19. ((∃x)Fx & (∃x)Gx) → (∃x)(Fx & Gx)
20. (∃x)(Fx & Gx) v (∃x)-Fx v (∃x)(Fx & -Gx)
21. (∃x)(Fx & -Gx) & (∃x)(Gx & -Fx) & (∃x)(-Fx & -Gx)
22. (∃x)(Fx & Gx) v (∃x)(-Fx & Gx) v (x)-Gx

x is just a Thing

Remember

most contingent

IV. (For part B, do not give an 𝔍 or 𝔍' that assigns to 'H^2' either the empty relation that contains no element or the relation 𝔇 x 𝔇 that contains as element every ordered pair ⟨m, n⟩ of objects m and n in 𝔇.)

23. (x)-Hxx
24. (x)(y)(Hxy → Hyx)
25 (x)((y) Hxy → Hxx)
26. (x)(Hxx → (y) Hxy)
27. (x)(∃y) Hxy & (y)(∃x)-Hxy
28. (∃x)(y) Hxy & (∃y)(x)-Hxy
29. (x)(-(∃y)Hxy v (∃y) Hyx)
30. (x)((∃y)-Hxy v (∃y)Hxy)
31. (∃x)((y)-Hxy v (y)Hxy)
32. (x)(y)(Hxy → -Hyx) & (x)(∃y)Hxy
33. (x)(y)(z)((Hxy & Hyz) → Hxz) & (x)(-Hxx & (∃y)Hxy)
34. (x)(y)(Hxy → Hyx) → (x)(y) (Hyx → Hxy)
35. (x)(y)(z)(Hxy → -Hyz)
36. (∃x)(y)(Hxy ↔ -Hxx)
37. (x)(∃y)(z)(Hxz → Hxy)

(y) Hxy reads:
'every y is an H of x'

Philosophy 12B
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Homework

Validity, Unsatisfiability, Contingency

Exercise

A. For each $\mathcal{L}$-sentence ϕ below indicate whether it is valid (V), r, unsatisfiable (U), or contingent (C).

B. If ϕ is contingent, show this by giving a trim interpretation $\mathcal{J}$ for which it is $\mathcal{J}$-true and a trim interpretation $\mathcal{J}'$ for which it is $\mathcal{J}'$-false. Either use notions from daily life or use one of the domains $\mathcal{D} = \{1,2,\ldots\}$.

C. If ϕ is valid, show this by reasoning about one complete interpretation $\mathcal{J}$ in a way which is typical, i.e., equally applicable to any other complete interpretation. (It may be helpful to imagine or, for an $\mathcal{J}$ with a small domain $\mathcal{D}$, actually draw the reduction tree for ϕ under $\mathcal{J}$; this is analogous to what we do in geometry where in reasoning about triangles in general we often imagine or draw a particular one.) For the $\mathcal{J}$ being considered find an appropriate distinction of cases so that the different cases together exhaust all possibilities and so that for each of these different cases you can show that the $\mathcal{L}$-sentence (which is at the root of the reduction tree) is $\mathcal{J}$-true. It is important that the case distinction and the reasoning in each case work equally well for any other complete interpretation.

Prove an $\mathcal{J}$-sent. valid by reason about [illegible] redux tree

D. If ϕ is unsatisfiable, proceed as in C, except that for each of the cases distinguished you show that ϕ is $\mathcal{J}$-false.

Suggestions.

a. Sometimes the task at hand can be replaced by one that is simpler. For example, one may find an $\mathcal{L}$-sentence ψ that is simpler than the $\mathcal{L}$-sentence ϕ that is given and that is logically equivalent

e.g. Find.

to ϕ. Instead of finding an $\mathfrak{I}$ under which ϕ is true, it then suffices to find an $\mathfrak{I}$ under which ψ is true. Also, one may find an $\mathcal{L}$-sentence χ that is simpler than $-\phi$ and logically equivalent to $-\phi$. Instead of finding an $\mathfrak{I}'$ under which ϕ is false, it then suffices to find an $\mathfrak{I}'$ under which χ is true. use of foregoing

b. There is probably no substitute for having intuitions on what an $\mathcal{L}$-sentence "says". Sometimes these can be developed or strengthened by translating the $\mathcal{L}$-sentence into "quasi-English". For example, '$(x)(Fx \rightarrow Gx)$' thus translates into: All F's are G's. Also, '$(x)((\exists y)Hyx \rightarrow (z)Hxz)$' becomes: For everything (it is that case that) if something is H-related to it then it is H-related to everything. ✓

c. There are various other crutches for intuition. Circles have often been used to represent sets that are associated with 1-ary predicates. For an ordered pair $\langle m,n \rangle$ the following has sometimes been used:

$m \longrightarrow n$ A binary relation can then be represented by certain points together with certain arrows between them.

A trick: employ a small Domain

d. Under B, if some $\mathfrak{D} = \langle 1,2,\ldots \rangle$ is used, it will save time to employ a small $\mathfrak{D}$. Some times $\mathfrak{D} = \langle 1 \rangle$ will do. At other times it won't, but $\mathfrak{D} = \{1,2\}$ will suffice. There are a few cases, however, where nothing smaller than $\mathfrak{D} = \langle 1,2,3 \rangle$ will work, and there is one case where one needs an infinite $\mathfrak{D} = \{1,2,\ldots, n, n+1, \ldots\}$.